



- Answer all the following question
- Illustrate your answers with sketches when necessary.
- The exam consists of **Three** page
- No. of questions: 3
- Total Mark: 90 Marks
- Examiner: Dr. Michael Nasief

Question (1): Signal Manipulation: [30 Marks]

A. For the signal shown in figure (1) draw: **[10 Marks]**

- The signal delayed 2 sec, attenuated by 2 and compressed by 2.
- The signal DC shifted by (-1), advanced by 1 and expanded by 2.

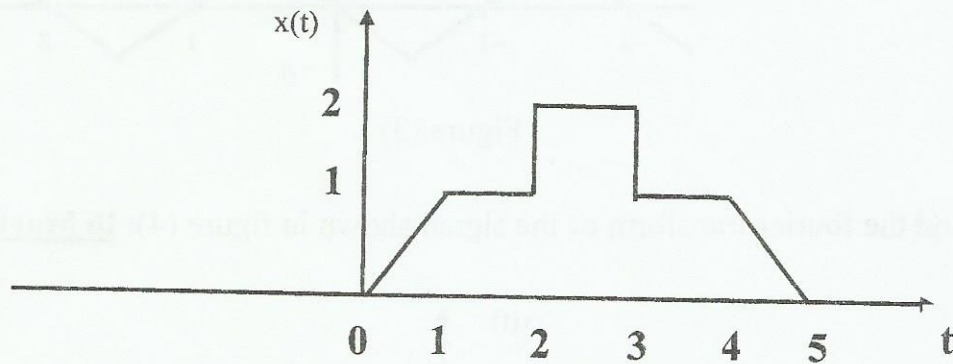


Figure (1)

B. Sketch and label the even and odd components of the signal shown in figure (1). **[8 Marks]**

C. For the signal $x(t)$ shown in figure (1) find the size of: **[6 Marks]**

- $x(t) [u(t) - u(t-1)]$.
- $x(t) [u(t-1) - u(t-2)]$.

D. Show that the following system is Time-Invariant/Time-varying: **[3 Marks]**

• $y(t) = \sin(t) x(t-2)$.

E. Show that the following system is Invertible or not: **[3 Marks]**

• $y(t) = 3 t x(t-2) - 2$.

Question (2): Convolution and Fourier: [30 Marks]

A. Convolve the signals shown in figure(2): **[8 Marks]**

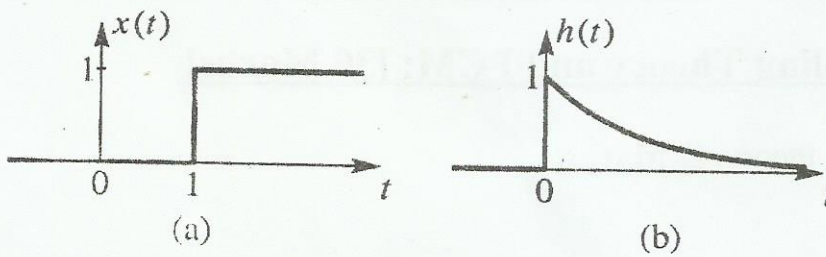


Figure (2)

Where:

$$x(t) = u(t-1).$$

$$h(t) = e^{-t} u(t).$$

B. Use direct integration to find expression for: **[4 Marks]**

$$y(t) = e^{-at} u(t) * e^{-bt} u(t).$$

C. Find the trigonometric Fourier series and compact trigonometric Fourier series for the triangular periodic signal shown in Fig. 3 over the interval $-1 \leq t \leq 1$. **[8 Marks]**

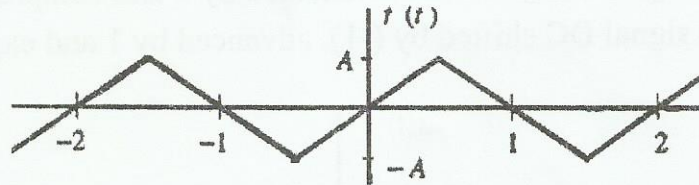


Figure (3)

D. Find the fourier transform of the signal shown in figure (4): **[6 Marks]**

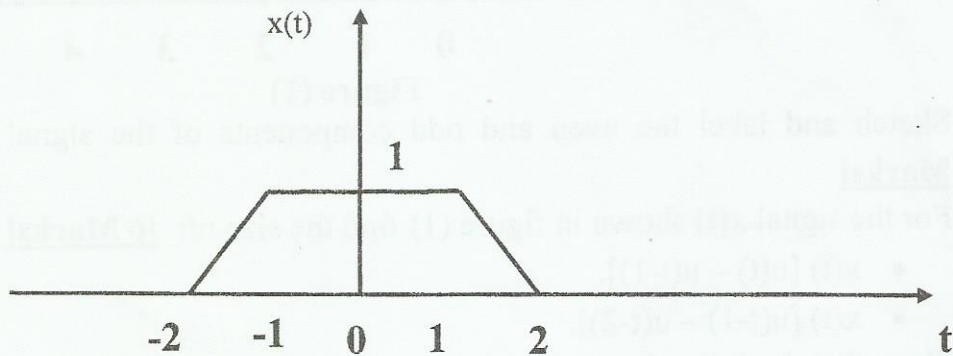


Figure (4)

E. Use Harmonics Tabular Method to find the Fourier series coefficients for the signal samples shown in the next table (get only the Dc component and the first harmonic coefficients): **[4 Marks]**

$V(t)$	10	20	30	35	40	40	35	30	20	10	5	0
wt	30	60	90	120	150	180	210	240	270	300	330	360

Question (3): Sampling Theory and PCM: [30 Marks]

A. State the sampling theory. **[2 Marks]**

B. Proof (mathematically) that the output spectrum of the sampled signal will be a duplicated version of the original spectrum. **[8 Marks]**

C. Define the Aliasing problem and show how you can avoid it. **[4 Marks]**

D. If $x(t)$ is the input signal to PCM encoder (Sound card in your PC): **[8 Marks]**

$x(t)$ = speech signal with peak to peak (5 v)

Find:

- Sampling frequency at Nyquist rate.
- If the step voltage between 2 consecutive quantum levels (Δv) = 10 mv:
 - What is the number of levels?
 - What is the number of bits per sample?
 - What is the bit rate?

E. Explain the PCM Encoder and Decoder **[6 Marks]**.

F. In your opinion what are the advantages and disadvantages of Delta Modulation? **[2 Marks]**

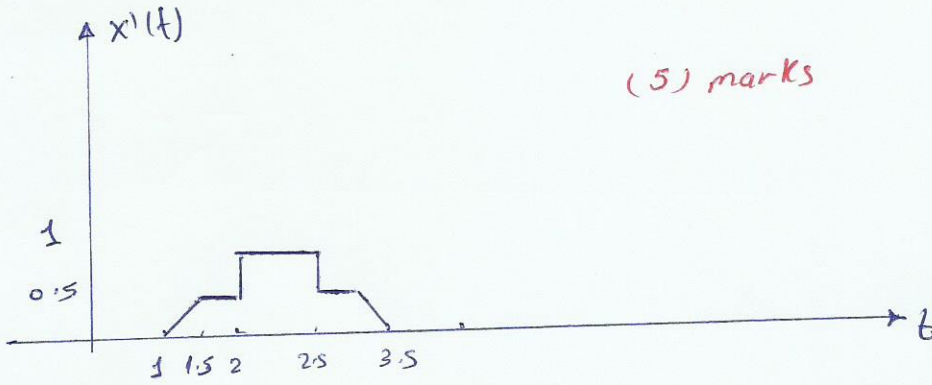
GOOD LUCK

DR. MICHAEL NASIEF

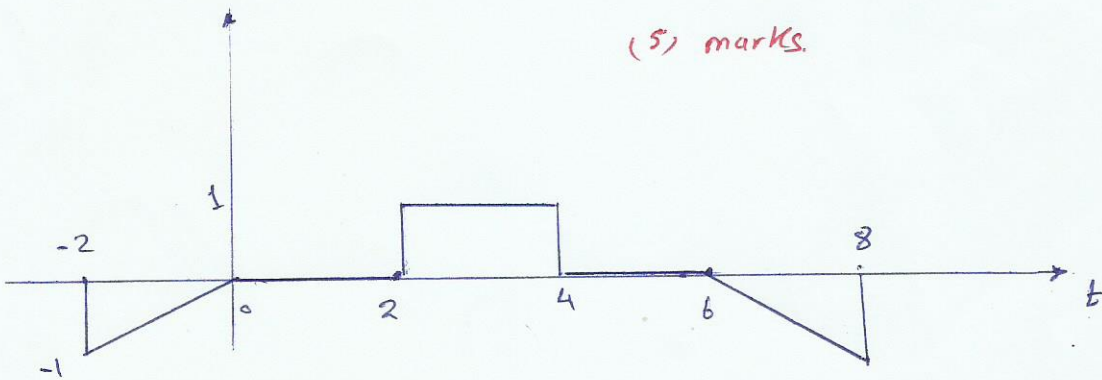
Q(1) Signal Manipulation [30 marks]

1

(A)

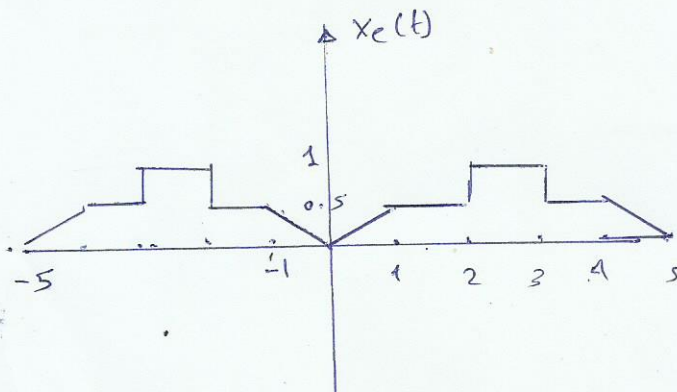


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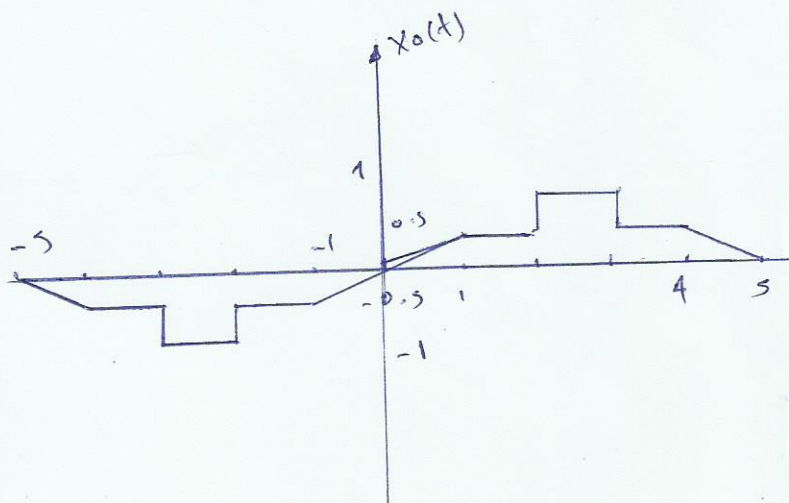
(B) $X_e = \frac{1}{2} x(t) + \frac{1}{2} x(-t)$

(4) marks



$X_o = \frac{1}{2} x(t) - \frac{1}{2} x(-t)$

(4) marks



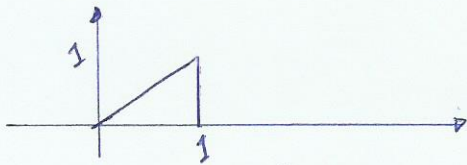
(C)

[2]

$$x(t) [u(t) - u(t-1)]$$

(3) marks

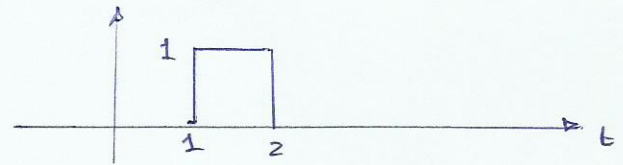
Energy signal



$$\int_{-\infty}^{\infty} x^2(t) dt = \int_0^1 t^2 dt = \frac{t^3}{3} \Big|_0^1$$
$$= \frac{1}{3}$$

$$x(t) [u(t-1) - u(t-2)]$$

Energy signal (3) marks.



$$\int_1^2 1^2 dt = 2 - 1 = 1$$

(D) Time varying system.

(3) marks

$$y_1(t) = x(t-t_0-2) \sin t$$

$$y_2(t) = x(t-t_0-2) \sin(t-t_0)$$

$y_1 \neq y_2 \therefore$ time varying system.

(E) The sys is invertible

(3) marks.

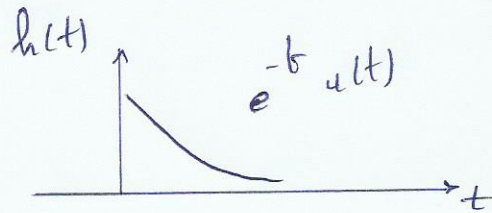
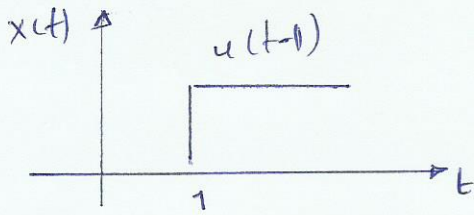
except for $t=0$

"we can say generally non-inv"

Q(2) Convolution and Fourier. [30 marks]

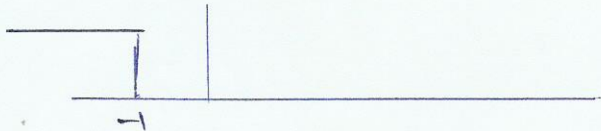
[3]

A



$x(-\tau)$

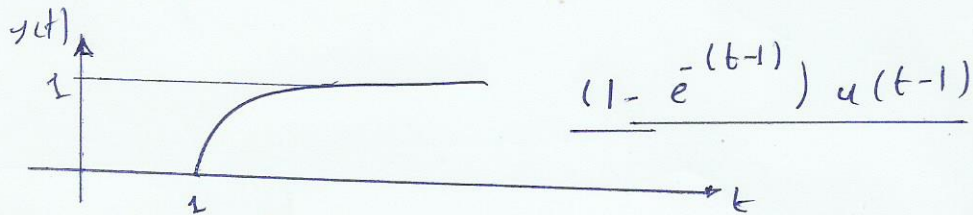
(8) marks



at $t-1 < 0$ no overlap $\therefore = 0$

at $t-1 \geq 0$

$$\int_0^{t-1} e^{-\tau} d\tau = -e^{-\tau} \Big|_0^{t-1} = 1 - e^{-(t-1)} \quad t \geq 1$$



B

$$y(t) = e^{-at} u(t) * e^{-bt} u(t)$$

(4) marks

$$= \int_{-\infty}^{\infty} e^{-a\tau} u(\tau) e^{-b(t-\tau)} u(t-\tau) d\tau$$

$$= \frac{e^{-at} - e^{-bt}}{b-a} u(t) \quad t > 0$$

$t < 0$



Period $T_0 = 2$

(8) marks

$$\omega_0 = \frac{2\pi}{T_0} = \pi$$

$$F(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\pi t + b_n \sin n\pi t$$

$$F(t) = \begin{cases} 2At & |t| \leq \frac{1}{2} \\ 2A(1-t) & \frac{1}{2} \leq t \leq \frac{3}{2} \end{cases}$$

choose interval $-\frac{1}{2}$ to $\frac{3}{2}$

$$a_n = \frac{2}{2} \int_{-\frac{1}{2}}^{\frac{3}{2}} f(t) \cos n\pi t dt$$

$$= 0$$

$$b_n = \int_{-\frac{1}{2}}^{\frac{1}{2}} 2At \sin n\pi t dt + \int_{\frac{1}{2}}^{\frac{3}{2}} 2A(1-t) \sin n\pi t dt$$

$$b_n = \frac{8A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right)$$

#

$$c_n = \sqrt{a_n^2 + b_n^2} = b_n \quad \text{since } a_n = 0$$



D

5

$$\mathcal{F}\left(\frac{d^2 x(t)}{dt^2}\right) = \frac{1}{2-1} [\delta(t+2) - \delta(t+1) - \delta(t-1) + \delta(t-2)]$$

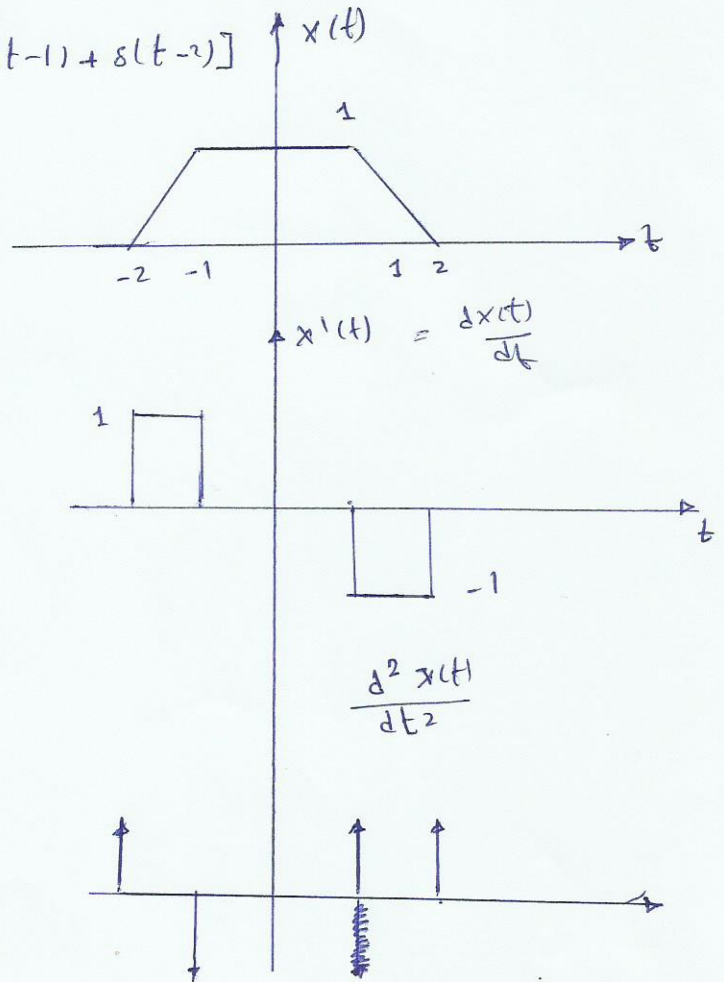
$$-\omega^2 F(\omega) = \frac{1}{2-1} [e^{j\omega 2} - e^{j\omega} - e^{-j\omega} + e^{-j\omega 2}]$$

$$= [\cos 2\omega - \cos \omega]$$

$$-\omega^2 F(\omega) = \cos 2\omega - \cos \omega$$

$$\therefore F(\omega) = -\frac{(\cos 2\omega - \cos \omega)}{\omega^2}$$

(6) marks



E

(4) marks

$$a_0 = \frac{275}{12} = 22.91$$

$$a_1 = \frac{-109.5}{6} = -18.25$$

$$b_1 = \frac{28.173}{6} = 4.695$$

Q(3) Sampling Theory

[6]

30 marks

(A) A cont. time signal $x(t)$ with freq. no higher than f_{max} Hz can be reconstructed exactly from its samples $x[n] = x(nT_s)$ if its samples taken at a rate $F_s = \frac{1}{T_s} \geq 2f_{max}$

(2) marks

(B) The sampled version can be expressed as:

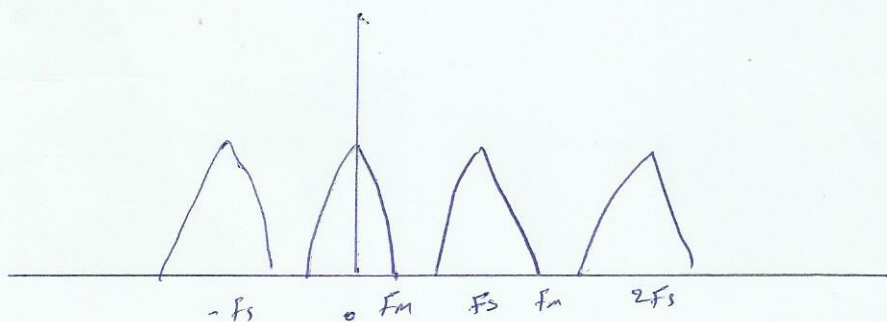
(8) marks

$$\bar{x}(t) = x(t) \delta_{T_s}(t) = \sum_n x(nT_s) \delta(t - nT_s)$$

$$\delta_n(t) = \frac{1}{T_s} [1 + 2\cos\omega_s t + 2\cos 2\omega_s t + \dots] \quad \omega_s = \frac{2\pi}{T_s}$$

$$\text{Since } 2\cos\omega_s t \Leftrightarrow X(\omega - \omega_s) + X(\omega + \omega_s)$$

$$\therefore \bar{X}(\omega) = \frac{1}{T_s} \sum_{-\infty}^{\infty} X(\omega - \omega_s)$$



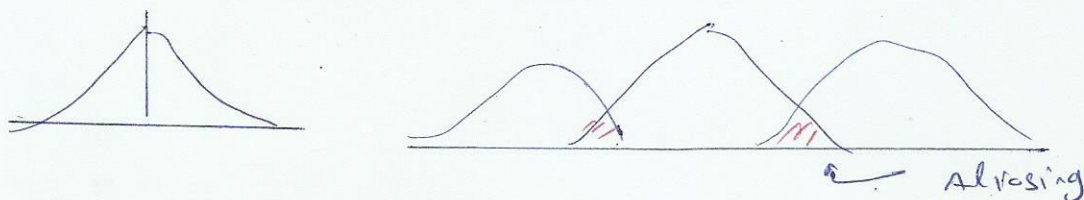
(C) Aliasing problem

(4) marks.

is the corruption of the original signal due to

$$B < f_s/2$$

after sampling



to avoid aliasing problem apply Low pass filter before
Sampling

D

$x(t)$ speech \therefore 4 kHz (8) marks

* $F_s = 2 F_m = 8 \text{ K Samples / sec}$

* $L = \frac{5 \text{ V}}{10 \text{ mV}} = 500 \rightarrow 512 \text{ levels}$

* $n = \log_2 L = 9$

* $F_b = n F_s = 9 \times 8 \text{ K} = 72 \text{ kbps}$

E

1) Sampling : $F_s \geq 2 F_m$ (6) marks

2) Quantizing : to the nearest quantum level

3) Encoding : convert to 0,1 $L = 2^n$

F

Adv
Low bit rate

(2) marks

disadv
Low quality